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ABSTRACT

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The Effects of Calculator Versus Conventional Algorithms for Fractions on Community College Student Computation, Understanding, and Attitude Scores

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Abstract

Three different methods of teaching fraction computation were compared using community college student scores on measures of fraction computation, fraction understanding, and attitude towards mathematics. One method used conventional algorithms; the second, a control for the effect of using a calculator, used conventional algorithms and calculators; the third used calculator-based algorithms. Differences between methods on the three dependent variables were not statistically significant. Pretest to posttest differences were significant for all groups with fraction computation scores making the largest contribution to the increase in scores.

The Effects of Calculator Versus Conventional Algorithms for Fractions on Community College Student Computation, Understanding, and Attitude Scores

Learning to perform calculations with positive rational numbers in common fraction form is not an easy task. In a summary of results from the first National Assessment of Educational Progress in mathematics, Carpenter, Coburn, Reys, and Wilson (1975) cite low performance on common fraction exercises as evidence that understanding and operating with fractions is intrinsically difficult. The belief that the algorithms for computation with positive rational numbers in decimal fraction form pose less difficulty than algorithms for positive rational numbers in common fraction form has led to experimentation with reversing the common fraction-decimal fraction instructional sequence. Johnson (1956) examined fraction computation in which common fractions were first converted to decimal fractions based on memorized equivalents. The increasing availability of hand-held calculators has eliminated the need for memorizing decimal equivalents of common fractions as well as paper and pencil computation with the decimal equivalents. Gaslin (1975) found at the school in which he was the instructor that ninth-grade students computed with common fractions significantly better using calculator-based algorithms than traditional algorithms. The calculator-based algorithms involved converting each common fraction to decimal fraction form on a calculator (set to truncate to thousandths) before performing the indicated operations.

In recent years, basic mathematics courses have proliferated for the estimated 10% to 25% of entering community college students who lack the prerequisite skills for college study (Losak, 1973). One topic emphasized in such courses is computational algorithms for common fractions. The present study was proposed after consideration of the needs of this group of mathematically underprepared community college students for quality instruction, the current interest in integrating calculators into classrooms, and Gaslin's assertion that for fraction computation calculator-based algorithms appear to be a "viable alternative" to conventional algorithms.

Problem

Three different methods of teaching fraction computation were designed for this study based on Gaslin's study. The problem was to determine their relative effectiveness.

In the first method of teaching fraction computation (T_1), a conventional algorithm set was used in which computation was performed according to "usual" text approaches. For example, to add 1/2 and 1/5, the student was instructed to find a common denominator, tenths, to change 1/2 to 5/10 and 1/5 to 2/10, to add numerators to obtain the sum 7/10, and, finally, to reduce the result if possible.

The second method of teaching fraction computation (T_2), was the same as the first except that students were instructed on how to use a calculator on whole number calculation. This method was included in the study as a control for the effect of learning to use a calculator.

The third method of teaching fractions (T_3) which was of main interest in this study involved an alternative algorithm set. In the alternative algorithm set, each fractional operand was converted on a calculator to a decimal truncated to thousandths. The indicated operation was then performed on the numbers in decimal form on the calculator. For example, to add 1/2 and 1/5, the student was instructed to convert the fractions to .5 and .2 on the calculator and to add the decimal values to obtain the sum of .7. For a fraction such as 1/3, it was noted that the calculator gave .33333333, only 8 digits of an unending number of digits. This decimal representation was to be truncated to thousandths prior to calculation. Conventional algorithms differ depending on the operation. The student has to distinguish which operation involves finding a common denominator, which involves inverting the divisor, and that denominators are not added in addition, but are multiplied in multiplication as well as remembering numerous steps such as reducing to lowest terms or "cancelling" common factors before multiplication. In contrast, each calculator algorithm is similar. Once the fractions are in decimal form, simply add, subtract, multiply, or divide as indicated.

To determine the relative effectiveness of the three methods of teaching fraction computation, measures of fraction computation skill, fraction understanding, and attitude towards mathematics were taken on three treatment groups of community college students before and after they were exposed to the different instructional methods. The instruments used were the Common Fraction Understanding and Common Fraction Computation subtests of the Stanford Diagnostic Arithmetic Test (SDAT)

(Buros, 1975, p. 369-371) and the Aiken Revised Math Attitude Scale (Aiken, 1963). The following research questions were investigated:

1. Are there differences among the three treatment groups on mean scores for fraction computation, fraction understanding, and attitude towards mathematics?

2. Are there any differences between pretest and posttest scores for fraction computation, fraction understanding, and attitude towards mathematics?

An alpha level of .05 was set for all statistical tests.

Method

Treatment Materials and Instruments

Nine instructional units on fractions and decimals were used. They are part of 39 units in a slide-audiotape-workbook presentation of arithmetic entitled Modumath. Permission had been obtained from the New York Community College system where the materials were developed to duplicate the materials. Since then, the workbook has been published (Hecht & Hecht, 1978). Five experimental units involving a calculator were constructed by using a format similar to Modumath materials and the same numerical examples as were contained in the Modumath units. In these units, calculator-based algorithms were substituted for the conventional algorithms found in the Modumath units. All answers to experimental unit self tests, exercises, and unit posttests were expressed in decimal form. The slide tape presentation for the experimental unit, Operating a Calculator, began with the idea that a calculator can save time and work. It, then, led the student through how to perform whole number addition, subtraction, multiplication and division on

a calculator as well as how to clear incorrect numeric and function key entries. One of the division examples resulted in a non-terminating decimal answer. Following the slide tape presentation students worked a variety of calculator exercises in a workbook followed by a self-scored unit posttest.

The instrument used to measure attitude was the Aiken Revised Math Attitude Scale (Aiken, 1963). It is a Likert-type scale with a range of 80 points and is considered reliable with r = .94 on test-retest comparison and was chosen for the study to be used as a general measure of math attitude.

The SDAT subtest of Common Fraction Understanding consists of 22 items. Four items concern naming a fractional part of a geometric figure or of an array of dots. Four items ask the student to change a fraction to higher terms or convert a mixed numeral to an improper fraction. Eight items ask questions about locating fractions as points on a number line. Six items ask that proper, improper and mixed numeral fractions be reduced to lowest terms. These skills were taught in the Modumath materials common to all treatments. This subtest was selected, however, as a measure of whether using calculator-based algorithms in which these concepts are not needed has an effect on the learning of these concepts.

The SDAT subtest of Common Fraction Computation consists of 28 items, eight items on adding fractions, eight items on subtracting fractions, six items on multiplying fractions, and six items on dividing fractions. Students are asked to compute the answer to a problem such as 6 1/3 - 4 4/5, reduce the answer to simplest terms, and write down

the answer. The test covers a variety of skills related to conventional fraction algorithms such as addition when neither denominator is common, subtraction of mixed numbers where neither denominator is common and borrowing is required, multiplication of a whole number times a fraction in mixed form, and division of a fraction by a whole number. The test was selected as being comprehensive of various fractional computation skills. The test is not multiple choice. For T_1 and T_2, half credit for a problem was deducted if answers were not reduced to simplest terms. For T_3, answers were expressed in decimal form. No test reliability information was computed for the sample in this study. However, the two subtests of the SDAT have reported split-half reliability coefficients ranging from .90 to .95 on other samples.

Subjects

The students enrolled in Basic Mathematics I (DVS 0710) at Walters State Community College in Morristown, Tennessee, were invited to participate. All 72 voluntarily signed consent forms. These students were enrolled in six different class sections. The experimenter taught four sections with a total of 41 students; a second teacher taught two sections with a total of 31 students. Scores on the Adult Basic Learning Examination, (ABLE), Level III subtest of arithmetic computation were used to assign students to treatment groups. First, student scores were ranked from highest to lowest. Then, the top three students were randomly assigned to the treatment groups followed by the next three students until all students were assigned. In this way, it was likely that each treatment group would contain students with similar arithmetic computation skill as measured by the ABLE.

Procedures

The three treatment groups were labeled as follows:

T_1: Conventional algorithm set without calculators (the traditional instruction method).

T_2: Conventional algorithm set with calculators (a treatment set up to control for the effect of learning to use a calculator).

T_3: Alternative algorithm set with calculators (the treatment of experimental interest).

All students were given a sheet instructing them to complete the first course module on whole numbers. Each then followed the normal classroom procedure of reading a summary of each unit in the module and taking and grading the unit self-test. If the student achieved the recommended score, he or she proceeded to the next unit. If not, the student viewed slides and listened to an audiotape for the unit. Following the tape, the student worked problems in the workbook and took and graded the unit posttest before proceeding to the next unit. The student completed the module after taking two teacher-made and teacher-graded exams, one covering the first half and another covering the second half of the module. Students scoring below 80% restudied and retook alternate forms of the exams until 80% was achieved.

Following the module on whole numbers, students entered the treatment phase of the experiment. Students were pretested with the two subtests of the SDAT, Form W, and the Aiken attitude scale. No calculators were used on the pretests. All treatment groups studied the following units:

Introducing Fractions

Re-naming Fractions

Understanding Decimal Fractions

Changing Fractions to Decimals

Adding Fractions with the Same Denominator

Adding Fractions with Different Denominators

Subtracting Fractions

Multiplying Fractions

Dividing Fractions

Then, students were posttested with the two subtests of the SDAT, Form X, and the Aiken attitude scale. Students in T_3 used calculators on the SDAT posttests.

Instructions on the sequence of study were contained in instruction sheets corresponding to the different treatments. Treatment T_1 consisted of the completion of the previously listed sequence without the use of calculators. Treatment T_2 consisted of the completion of the same sequence except that immediately after the pretests, students completed a unit entitled Operating a Calculator, and then were permitted to use a calculator any time they wanted. Use of a calculator was of little benefit in T_2, however, except on the unit, Changing Fractions to Decimals. Thus treatments T_1 and T_2 were taught the conventional algorithms for fraction computation, with T_2 using a calculator on two units.

Treatment T_3 involved the calculator-based algorithms for fraction computation. Students in this treatment followed the previously listed

sequence of units with Operating a Calculator inserted as it was in T_2 just after the pretest. Then students in T_3 used the following calculator-based units instead of the conventional algorithm units of Modumath:

Adding Fractions with Different Denominators

Subtracting Fractions

Multiplying Fractions

Dividing Fractions

The decision was made to have students in T_3 study the regular Modumath unit on Adding Fractions with the Same Denominator. The initial instruction in the slide tape presentation of the experimental unit on Adding Fractions with Different Denominators begins with adding 1/4 + 1/3 using the conventional algorithm just as the regular Modumath unit does. The difficulty of the algorithm is contrasted with the simplicity of converting each fraction to a decimal (truncated) and then adding. The remainder of the unit and the three units following then use the calculator-based algorithms. This approach was taken because it was felt that all the students had been previously exposed to conventional algorithms in elementary and secondary school, but, as indicated by ABLE pretest scores, had not retained the algorithms.

In summary, the experiment consisted of pretesting during which calculators were not used; a unit on operating a calculator for treatments T_2 and T_3 during which calculators were used; nine units on fractions and decimals, four of which taught calculator-based algorithms and required calculator use by T_3 students; and posttesting during which calculators were used by T_3.

Results

To test the difference between treatments, a multivariate analysis of variance was used on computation, understanding and attitude scores. Since the same subjects were pretested and posttested, a repeated measures design was used. A multivariate instead of a univariate analysis was used for two reasons. First, multivariate tests account for correlations between variables which would positively bias univariate tests (Bock & Haggard, 1968, p. 102). (Intercorrelations were computed among the variables of computation, understanding, and attitude on pretest and posttest measures. Most intercorrelations were significantly different from zero; for example, the variables of pretest understanding and pretest computation had a correlation of .67). Second, a series of univariate tests to determine treatment differences on each dependent variable separately would inflate the Type I error (Harris, 1975).

Of the 72 students originally assigned to treatment groups, 16 stopped attending class, two took tests at the wrong time, and two failed to complete the treatment phase of the experiment by the end of the school term though they persisted in attendance. This loss of 20 students left 16 students in T_1 and T_2 and 20 in T_3. The decision was made to drop four randomly selected students from T_3 leaving 48 students, 16 per treatment group. The ABLE pretest scores were analyzed using analysis of variance to determine whether the loss of subjects had caused any treatment group to differ from the others on initial computational ability. There was no significant difference among treatment groups as measured by the ABLE.

Research Question One

This question deals with whether the three treatment groups differed in mean scores for fraction computation, fraction understanding, and attitude towards mathematics. Table 1 summarizes group means and standard deviations on pretest and posttest measures.

Table 1 about here

Table 2 gives the multivariate analysis of variance (MANOVA) summary. The multivariate test statistic is the Hotelling-Lawley trace which is transformed to an F value (Tatsuoka, 1971, p. 212).

Table 2 about here

Since the probability for Treatment (.18) was greater than $\alpha = .05$, it was concluded that the mean scores on the three dependent measures for the three treatment groups did not differ significantly.

Research Question Two

This question deals with whether there was a change from pretest to posttest scores for fraction computation, fraction understanding, and attitude towards mathematics. Since the probability for Exam (.0001) was less than $\alpha = .05$, it was concluded that posttest scores on the three dependent variables were significantly higher than pretest scores. When a significant difference is found in a MANOVA, a discriminant analysis is performed to determine the directions along which the major differences occurred (Tatsuoka, 1971, p. 157). The discriminant function with

standardized discriminant function coefficients was found to be $Y^* = 1.05X_1 - .06X_2 + .18X_3$. Computation, understanding, and attitude were represented by X_1, X_2, and X_3, respectively. According to Tatsuoka (1971, p. 163), the relative sizes of the coefficients without regard to sign, 1.05, .18, and .06, were indications that most of the gain from pretests to posttests could be accounted for by the dependent variable of fraction computation.

The third multivariate F has to do with interaction between instructional method and time of exam. The six mean vectors representing test performance on computation, understanding, and attitude for the three treatments under pretest and posttest conditions did not differ.

Limitations

The study was carried out in a classroom setting, so there was less control than would have been possible in a laboratory setting. Students were enrolled in a self-paced course. Therefore, they entered the treatment phase of the experiment at different times depending upon their completion of the module on whole numbers. In each class section, there were students who had been assigned to each of the three treatment groups. Treatment materials looked similar and were to be studied individually. Yet it was apparent to the students that some students were using calculators and others were not. Students had given their consent to participate in a study on learning fractions and that some students would be asked not to use calculators.

That numbers such as 1/3 and .333 are not equivalent was not discussed in the fraction units. A later unit in the course deals with rounding

numbers to a particular place value; and it was felt that the discussion could be postponed until then. The effect of rounding (or truncating) on accuracy is important to the application of the results of computation, yet it is not a simple concept to teach.

Discussion

The non-significant treatment differences with respect to fraction computation, fraction understanding, and attitude towards mathematics are not unusual in the light of general research on calculator effects on achievement and attitude (Note 1). The results of the present study do not support Gaslin's (1975) results. However, he worked with ninth-graders, intact classes, teacher-oriented instruction, desk calculators, and non-standardized dependent measures, all of which differed from the present study.

The significant pretest to posttest gain was an indication that in all three treatments students were able to learn fraction computation algorithms. According to the discriminant function analysis, most of the pretest to posttest gain could be attributed to computation scores. The mean percentage correct on the computation subtest for all 48 students increased from 41% to 80%. The mean length of treatment for all students was 22 days (SD=9). Thus, the calculator algorithms can serve as an effective alternative instructional strategy where computational skill is a goal of instruction. It should be noted, also, that the mean percentage correct on the understanding subtest for all 48 students increased from 60% to 77%, indicating some improvement, and that the students using calculator-based algorithms did not score less well on understanding than students using conventional algorithms. The treatments appeared

to have no effect on attitudes over the comparatively short treatment period.

Recommendations

One avenue for further research appears to be of some interest. The shortest calculator algorithms involve converting each common fraction to decimal form, writing each result on paper and then re-entering each decimal to perform the indicated operation. Some use of the calculator memory was incorporated in the instructional materials. The conventional algorithms cannot be simplified, but is is possible to present a simpler subset of calculator algorithms by eliminating the use of calculator memory. Whether shortening the calculator algorithms might decrease the length of instruction required to attain the same level of understanding and computational skill ought to be examined by future research since time is very important to the developmental studies student at the college level.

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TABLE 1

GROUP MEANS AND STANDARD DEVIATIONS OF SCORES

	Attitude				Understanding				Computation			
	Pre		Post		Pre		Post		Pre		Post	
	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
T_1	52.44	(13.03)	54.06	(12.57)	12.13	(5.10)	16.69	(3.42)	10.41	(9.80)	21.69	(6.92)
T_2	57.50	(16.18)	56.94	(15.58)	14.56	(5.09)	18.00	(3.98)	11.38	(8.46)	21.91	(5.68)
T_3	60.13	(22.36)	61.63	(18.91)	12.56	(6.00)	16.38	(4.11)	12.38	(10.22)	24.00	(4.32)

Note: n = 16 for each of T_1, T_2, T_3

TABLE 2

MULTIVARIATE ANALYSIS OF VARIANCE OF SCORES

Source	df	Multivariate F	Prob.>F
Treatment (T)	6, 84	1.52	.18
Exam (E)	3, 43	25.11	.0001
E x T	6, 84	.18	.98

Note: Description of Prob.> F: If the null hypothesis is true, the value listed in the Prob.> F column is the probability of obtaining an F ratio as large or larger than the ratio calculated for this sample. If the probability value is less than the chosen α, the null hypothesis is rejected.